M09/5/MATSD/SP2/ENG/TZ1/XX/M+

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MARKSCHEME

May 2009

MATHEMATICAL STUDIES

Standard Level

Paper 2

25 pages

Paper 2 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

1 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, irrespective of working shown.

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently, marks not awarded.

ft Marks that can be awarded as **follow through** from previous results in the question

In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**. Marks to be awarded for unsupported answers are designated ***G*** in the mark scheme as such answers will usually arise from working performed on a graphic display calculator.

2 Method of Marking

(a) All marking must be done using a **red** pen.

(b) Marks must be noted on candidates' scripts as in the markscheme:

- Show the breakdown of individual marks using the **abbreviations *(M1)***, ***(A2)*** *etc.*;
- Write down each part mark total, as indicated on the **question paper**. These totals should be written in the **margins** of the candidates' answer booklets;
- Write down and circle the total for each question at the end of the question.
- Transfer the total for **each question** to the front cover sheet and write down the total mark for the paper

(c) Working crossed out by the candidate should not be awarded any marks.

(d) Where candidates have written two solutions to a question, only the first solution should be marked.

(e) If correct working results in a correct answer but then further working is developed, full marks are **not** always awarded. In most such cases it will be a single final answer mark that is lost. Full marks **can** be awarded if the candidate demonstrates clear understanding of the task and the result. If in doubt, consult your team leader.

(f) Candidate drawn graphs will have a single ***(A1)*** available for scales and labels combined. This can be awarded if all these are present and correct, even if no graph is drawn, however, the mark should not be awarded if the scale shown is inappropriate to, or inadequate for, the required missing graph. In papers which have two candidate drawn graphs, consistent errors in showing labels or scales can follow through on the second graph, though not if the error is complete omission of these features.

Please note: Assignment of marks to the answers in all the following examples is for demonstration purposes only. Marks for actual examination questions will not necessarily follow the same pattern.

Question: Using Pythagoras to find a side of a triangle:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4}=\sqrt{13}$ ***(M1)(A1)*** (3.61 3s.f.) **OR** Answer only ***(G2)***	Case (i) $\sqrt{13}$ or 3.61 or both Case (ii) $\sqrt{9+4}=\sqrt{13}$ $= 6.50$	***(G2)*** ***(M1)*** ***(A0)***

Question: Calculate the gradient of the line passing through the points (5,3) and (0,9).

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5}=-\frac{6}{5}$ ***(M1)(A1)*** **OR** Answer only ***(G1)***	(i) –6/5 (ii) $\frac{9-3}{0-5}=-\frac{6}{5}$ Gradient is -6/5 $y = -6x/5 + 9$ (iii) $\frac{9-3}{0-5}=-\frac{6}{5}$ $y = -6x/5 + 9$	***(G1)*** ***(M1)*** ***(A1)*** *(There is clear understanding of the gradient.)* ***(M1)*** ***(A0)*** *(There is confusion about what is required.)*

Question: sine rule used to find angle *A*, with angle *B* and side *b* known but side *a* is first calculated using Pythagoras in an adjoining triangle.

Markscheme		Candidate's Script		Marking
$a=\sqrt{25+36}=\sqrt{61}$	***(M1)(A1)***	Case (i)	$a=\sqrt{61}$	***(G2)***
OR answer only	***(G2)***			
$\frac{\sin(A)}{\sqrt{61}}=\frac{\sin(32)}{5}$	***(M1)(A1)***		$A=55.9^\circ$	***(A2)***
$A=55.9^\circ$	***(A1)***	Case (ii)	$A=55.9^\circ$ *(with no mention of a)*	***(A2)***
OR answer only	***(A2)***			

3 Follow-through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'** appended to the eligible mark(s).

- If an answer resulting from follow through is extremely unrealistic (*e.g.* negative distances or wrong by a large order of magnitude) then the final ***A*** mark should not be awarded. If in doubt, contact your team leader.

- If a question is transformed by an error into a **different**, **much simpler question** then follow through might not apply or might be reduced. In this situation consult your team leader and record the decision on the candidate's script.

- To award follow through marks for a question part, **there must be working present for that part** and not just an answer based on the follow through. An isolated follow through answer, with no working, must be regarded as incorrect and receives no marks **even if it seems approximately correct**.

- Inadvertent use of radians will be penalised the first time it occurs. Subsequent use, even in later questions will normally be allowed follow through marks unless the answer is unrealistic. Cases of this kind will be addressed on an individual basis.

Implementation: The following examples illustrate correct use of the **follow through** process in straightforward situations.

Question: An investment problem with two different rates of interest and a total amount of $600 split across the rates in consecutive periods:

Markscheme	Candidate's Script	Marking
(a) $ 600 × 1.02 ***(M1)***	Case (i)	
= $ 612 ***(A1)***	(a) Final amount after 1st period	
	= $ 600 × 1.02	***(M1)***
OR answer only ***(G2)***	= $ 602	***(A0)***
(b) $\$\left(\frac{612}{2}\times 1.02\right)+\left(\frac{612}{2}\times 1.04\right)$ ***(M1)***	(b) Amount after 2nd period	
= $ 630.36 ***(A1)*(ft)**	= 301 × 1.02 + 301 × 1.04	***(M1)***
	= $ 620.06	***(A1)*(ft)**
OR answer only ***(G1)***		
Note: The (M1) is for splitting the value from (a) and forming a sum of products.	but note Case (ii) an ***(M0)*** almost always prohibits the **associated (ft)** so	
Here the **(ft)** indicates a possible follow through from part (a).	(a) $ 600 × 1.02 = $ 602	***(M1)(A0)***
	(b) $ 602 × 1.04 = $626.08	***(M0)(A0)*(ft)**
	Case (iii)	
	(a) $ 600 × 1.02 = $ 602	***(M1)(A0)***
	(b) No working. 620.06 given as answer.	***(G0)*(ft)**
	Case (iv)	
	(a) $ 612	***(G2)***
	(b) $ 630.36	***(G1)***

Question: Using trigonometry to calculate angles and sides of triangles.

Markscheme	Candidate's Script	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)*** $A = 22.0^\circ$ ***(A1)***	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$	***(M1) (A0) (use of sine rule but with wrong values)***
OR answer only ***(A2)***	$A = 41.8^\circ$	***(A0)*** *(Note: the 2nd* ***(A1)*** *here was not marked* **(ft)** *and cannot be awarded, because there was an earlier error in the same question part.)*
(b) $x = 7 \tan A$ ***(M1)*** $= 2.83$ ***(A1)*****(ft)**	(b) case (i) $x = 7 \tan A$ $= 6.26$	***(M1) (A1)*****(ft)**
OR answer 2.83 only ***(G1)***	***but*** case (ii) 6.26	***(G0)***

4 Using the Markscheme

This markscheme presents a particular way in which each question might be worked and how it should be marked.

(a) As ***A*** marks are normally **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark, unless otherwise instructed in the markscheme. (See the finance example above).

Similarly ***(A1)(R0)*** cannot be awarded for an answer which is accidentally correct for the wrong reasons given.

Example: Question: (a) χ^2 calculated followed by (b) degrees of freedom found and (c) and (d) comparison to critical value. (Dependence of ***A*** and ***R*** marks.)

Markscheme	Candidate's Script	Marking
	Case (i)	
(a) $\chi_{calc}^{2} = 3.92$ ***(A1)***	(a) $\chi_{calc}^{2} = 3.92$	***(A1)***
(b) $n = 4$ ***(A1)***	(b) $n = 4$	***(A1)***
(c) $\chi_{crit}^{2} = 9.488$ ***(A1)*****(ft)**	(c) Don't know?	***(A0)***
(d) Do not reject null hypothesis ***(A1)*****(ft)** because $\chi_{calc}^{2} < \chi_{crit}^{2}$ ***(R1)*****(ft)**	(d) Do not reject null hypothesis because $\chi_{calc}^{2} > 0$	***(A0)*****(ft)** ***(R0)*****(ft)** *(**(A0)** was awarded here because the reason is wrong.)*

	Case (ii) (a) $\chi_{calc}^{2} = 3.92$	***(A1)***
	(b) $n = 4$	***(A1)***
	(c) $\chi_{crit}^{2} = 4.488$	***(A0)***
	(d) Do not reject null hypothesis because $\chi_{calc}^{2} < \chi_{crit}^{2}$	***(A1)*****(ft)** ***(R1)*****(ft)**
	Case (iii) (a) $\chi_{calc}^{2} = 3.92$	***(A1)***
	(b) n = 1	***(A0)***
	(c) $\chi_{crit}^{2} = 3.841$	***(A1)*****(ft)**
	(d) Reject null hypothesis because $\chi_{calc}^{2} > \chi_{crit}^{2}$	***(A1)*****(ft)** ***(R1)*****(ft)**

(b) **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner that is consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*. This includes alternatives obtained with a graphic display calculator. In such cases, alternative ***G*** mark assignments for answer only will not be repeated if this is redundant.

Example: Question to find the coordinates of a vertex of a given quadratic.

Working	Marks
$f(x) = 2x^2 + 7x - 3$	
$x = -\frac{b}{2a} = -\frac{7}{4}$ ***(M1)*** *for use of – b/2a,* ***(A1)*** *for correct answer*	***(M1)(A1)*** **or** ***(G2)***
$f(-7/4) = -\frac{146}{16} = -\frac{73}{8}$	***(M1)(A1)*****(ft)**
(M1) *for using f(-7/4),* ***(A1)*** *for answer.*	**or** ***(G1)***
Coordinates are (–7/4, –73/8)	***(A1)*****(ft)**
OR	**OR**

(–7/4, –73/8) (with no working at all)	***(G2)(G1)***
OR	**OR**
$f'(x) = 4x + 7$, $4x+7 = 0$	***(M1)***
so $x = -7/4$ ***(M1)*** *for attempting to take a derivative and setting it to 0* ***(A1)*** *for answer*	***(A1)***
$f(-7/4) = -\frac{146}{16} = -\frac{73}{8}$ ***(M1)*** *for using f(-7/4),* ***(A1)*** *for answer.*	***(M1)(A1)*****(ft)**
Coordinates are (–7/4, –73/8)	***(A1)*****(ft)**

(c) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

(d) As this is an international examination, all valid **alternative forms of notation** should be accepted.

Some examples of these are:

Decimal points: 1.7; 1'7; $1 \cdot 7$; 1,7.

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*e.g.* complement): A'; $\bar{A}$; A^c; $U - A$; (*A*

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.
$p \Rightarrow q$; $p \to q$; $q \Leftarrow p$.

(e) Discretionary ***(d)*** marks: There will be rare occasions where the markscheme does not cover the work seen. In such cases, ***(d)*** should be used to indicate where an examiner has used discretion. It must be accompanied by a brief note to explain the decision made.

5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to 3 significant figures.

A penalty known as an **ACCURACY PENALTY *(AP)*** is applied if an answer is either

(i) rounded incorrectly to 3 significant figures or

(ii) rounded correctly or incorrectly to some other level of accuracy.

This penalty is applied to the **final answer** of a question part only. It applies **also** when an exact answer is incorrectly rounded.

THE ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER! Subsequent accuracy errors can be **ignored** and full marks awarded if all else is correct.

An accuracy penalty must be recorded in proximity to the incorrect answer as ***(A0)(AP).***
Examiners must record the occurrence of an accuracy penalty by writing ***(AP)*** next to the relevant question total on the front of the cover sheet.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. This is **NOT** an accuracy penalty. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. If the rounding is incorrect, an accuracy penalty should be applied as detailed above. Exact answers such as $\frac{1}{4}$ can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of π and answers taking the form of square roots of integers **(even if exact squares)** or any rational power of an integer (*e.g.* $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}, \sqrt{9}$) may be accepted as exact answers. All other powers (*e.g*. of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

Answers **with no supporting working** which are written correct to more than 3 significant figures should be marked according to the scheme for correct answers with no working, but with an ***(AP)*** then applied. When this happens, **(A2)** or **(G2)** can be split if necessary (*e.g.* ***(A1)(A0)(AP)*** or ***(G1)(G0)(AP)***). If there is no working shown, and answers are given to the correct two significant figures, apply the ***(AP)***. However, do not accept answers to one significant figure without working.

An accuracy penalty should not be applied to an answer that is already incorrect for some other reason.

Special cases

An answer taken directly from the IB chi-squared statistical table can be given and used to the same level of accuracy as appears in the table (3 decimal places) or correct to 3 significant figures.

For judging equivalence between 3 significant figures and use of minutes and seconds for angles, guidelines have been issued to paper setters. This problem will be dealt with on an individual basis as the need arises.

Examples: The Pythagoras example used before:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4}=\sqrt{13}$ ***(M1)(A1)*** (3.61 3s.f.) **OR** answer only ***(G2)***	(i) 3.6 or 4	***(G0)***
	(ii) 3.60555	***(G1)(G0)(AP)***
	(iii) $\sqrt{9+4}=\sqrt{13}$	***(M1)***
	= 3.6	***(A0)(AP)***
	(iv) $\sqrt{9+4}=\sqrt{13}$	***(M1)***
	= 3.60555	***(A0)(AP)***
	(v) $\sqrt{9+4}=\sqrt{13}=3.60$	***(M1)(A0)(AP)***
	(vi) $\sqrt{9+4}=\sqrt{14}=3.74$	***(M1)(A0)***

If the question specified *e.g.* correct to 4 decimal places for the answer, then there would be one extra mark available as follows:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4}=\sqrt{13}$ ***(M1)(A1)*** **OR** answer only ***(G2)*** *(Note: requires more than 4 d.p.)*	(i) 3.605551 = 3.6056 (4 d.p.)	***(G2)(A1)***
	(ii) $\sqrt{9+4}=\sqrt{13}$	***(M1)(A1)***
= 3.6056 (4 d.p.) ***(A1)*(ft)**	= 3.606	***(A0)***
OR answer only ***(G2)***	(iii) 3.60555	***(G2)(A0)***
OR answer 3.606 or 3.61 only ***(G1)***	(iv) 3.6056	***(G2)***
	(v) $\sqrt{9+4}=\sqrt{14}$	***(M1)(A0)***
	= 3.7417	***(A1)*(ft)**
	(vi) $\sqrt{9-4}=\sqrt{5}$	***(M0)(A0)***
	= 2.2361	***(A1)*(ft)** *(Note: this is a special case, where the initial **(M0)** does not determine the final **(A0)** because the correction to **4dp** is an entirely new task.)*
	(vii) 3.606	***(G1)***

Premature Rounding

Accuracy errors in a final answer, which result from premature rounding earlier in the same question part, should not receive an accuracy penalty. There are two situations. If there is a mark available for a prematurely rounded answer and the rounding occurs at this stage, then the inappropriate rounding should be penalised with ***(A0)*** but the answer can then be allowed to follow through to the end of the question. If the first stage of the answer is correct but rounded further on, then it should be penalised at an appropriate place close to where it is rounded. Some discretion should be used to deny a **(ft)** mark if the rounding is very bad and the answer far from its required value.

Example: Question: sine rule used to find angle *A*, with angle *B* and side *b* known but side *a* is first calculated using Pythagoras in an adjoining triangle.

Markscheme	Candidate's Script	Marking
$a=\sqrt{25+36}=\sqrt{61}$ ***(M1)(A1)***	(i) $a=\sqrt{25+36}=\sqrt{61}$ $= 7.8$	***(M1)*** ***(A0)***
OR answer only ***(G2)***		
$\frac{\sin(A)}{\sqrt{61}}=\frac{\sin(32)}{5}$ ***(M1)(A1)*(ft)**	$\frac{\sin(A)}{7.8}=\frac{\sin(32)}{5}$	***(M1)(A1)*(ft)**
$A = 55.9^\circ$ ***(A1)*(ft)**	$A = 55.8^\circ$	***(A1)*(ft)**
OR answer only ***(G2)***	(ii) $a=\sqrt{25+36}=\sqrt{61}$	***(M1)(A1)***
	$\frac{\sin(A)}{7.8}=\frac{\sin(32)}{5}$	***(M1)(A0)***
	$A = 55.8^\circ$	***(A1)*(ft)**
	(iii) $a=\sqrt{25+36}=\sqrt{61}$	***(M1)(A1)***
	$\frac{\sin(A)}{7.8}=\frac{\sin(32)}{5}$	***(M1)(A0)***
	$A = 55.9^\circ$	***(A0)(AP)*(ft)** *(even though this is the answer to the question, the rounded answer does not follow from the given working.)*
	(iv) $a=\sqrt{25+36}=\sqrt{61}$	***(M1)(A1)***
	$\frac{\sin(A)}{7.8}=\frac{\sin(32)}{5}$	***(M1)(A0)***
	$A = \sin^{-1}(0.83) = 56.1^\circ$	***(A0)***
	(v) $a=\sqrt{25+36}=\sqrt{61} = 8$	***(M1)(A0)***

	$\frac{\sin(A)}{8}=\frac{\sin(32)}{5}$	***(M1)(A1)*****(ft)**
	$A = 58.0^{\circ}$	***(A0)*****(ft)** *(The rounding is severe and the answer quite far from correct).*
	(vi) $a = 7.8$	***(G0)***
	$A = 55.8^{\circ}$	***(G0)*****(ft)** *(there is no working to justify the follow through.)*

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places, but could differ in rare instances depending on the currency in question.

A penalty known as a FINANCIAL ACCURACY PENALTY ***(FP)*** is applied if an answer does not adhere to the specification in the question. This penalty is applied to the final answer of a question part only.

THE FINANCIAL ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER! Subsequent financial accuracy errors can be ignored and full marks awarded if all else is correct.

A financial accuracy penalty must be recorded in proximity to the incorrect answer as ***(A0)(FP)***.
Examiners must record the occurrence of a financial accuracy penalty by writing ***(FP)*** next to the relevant question total on the front cover sheet.

The financial accuracy penalty is imposed only for rounding to the wrong level of accuracy and NOT for incorrect rounding to the required number of places. The latter would incur a normal accuracy penalty ***(AP)***.
No single answer can receive two penalties. If both types of error are present then *(FP)* takes priority.

Please see the examples below.

NOTE: The financial accuracy penalty will be flagged in the markscheme at the start of each answer where it could apply, with the words “Financial accuracy penalty ***(FP)*** is applicable in this question”. If this instruction is not present, then do not apply the penalty. An ***(FP)*** will also be present in the left hand column next to where it applies.

Example: A financial question demands accuracy correct to 2dp.
Prior to rounding the answer is $231.6189

Markscheme	Candidate's Script	Marking
Financial accuracy penalty ***(FP)*** is applicable in this question		
$231.62 ***(A1)***	$231.62 or 231.62	***(A1)*** (No unit penalty (see section 7 below) for missing $ symbol.)
	231.6 or 232	***(A0)(FP)*** (Correct rounding process but incorrect level.)
	231.61	***(A0)(AP)*** (Incorrect rounding process to correct level.)
	231	***(A0)(FP)*** (Both types of error occurred but ***(FP)*** takes priority.)
	232.00	***(A0)(AP)*** (It's not clear whether nearest dollar or 2dp was really intended but we interpret as 2dp rounded incorrectly.)

7 Units in answers

A penalty known as a UNIT PENALTY ***(UP)*** is applied if an answer does not include the correct units. This applies both to missing units and to incorrect units. This penalty is applied to the final answer of a question part only.

THE UNIT PENALTY IS APPLIED AT MOST ONCE PER PAPER! Subsequent unit errors can be ignored and full marks awarded if all else is correct.

A unit penalty must be recorded in proximity to the incorrect answer as ***(A0)(UP)***. Examiners must record the occurrence of a unit penalty by writing ***(UP)*** next to the relevant question total on the front cover sheet.

NOTE: The unit penalty will be flagged in the markscheme at the start of each answer where it could apply, with the words "Unit penalty ***(UP)*** is applicable in this question". If this instruction is not present, then do not apply the penalty. A ***(UP)*** will also be present in the left hand column next to where it applies.

NOTE: In this context, symbols for currency such as $ or GBP etc are not considered units. Candidates are encouraged to include them but should not be penalised if they are missing.
Missing degree symbols and percentage symbols are also not eligible for a unit penalty.

No single answer can receive two penalties. If an answer is rounded incorrectly and also has wrong or missing units, apply the accuracy penalty *(AP)* only. If the *(AP)* has already been used, such an answer is eligible for the unit penalty.

Example: A question has answer to part (i) of 66.2 cm. The answer before rounding is 66.213 cm.
Part (ii) involves dividing by 60 with units cm/s. Assume that the *(UP)* has not been used previously.

Markscheme	Candidate's Script	Marking
*Unit penalty **(UP)** is applicable in this question*		
(i) 66.2 cm ***(A1)***	(i) 66.2cm	***(A1)***
(ii) 1.10 cm/s ***(A1)***	(ii) 1.10 cm/s	***(A1)***
	(i) 66.2	***(A0)(UP)***
	(ii) 1.10	***(A1)***
	(i) 66.2 cm	***(A1)***
	(ii) 1.10	***(A0)(UP)***
	(i) 66	***(A0)(AP)*** if ***(AP)*** not used previously but ***(A0)(UP)*** otherwise.
	(ii) 1.1	***(A0)(UP)*** if ***(AP)*** used in part (i) but ***(A1)*****(ft)** for correct follow through to

		exact answer if ***(UP)*** used in part (i).
	(i) 66	***(A0)(AP)*** if ***(AP)*** not used previously but ***(A0)(UP)*** otherwise.
	(ii) 1.1 cm/s	***(A1)*(ft)**

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.

QUESTION 1 ***Unit penalty (UP) applies in parts (a)(i), (ii) and (e)(i)***

(UP) (a) (i) median $= 13$ seconds ***(A1)***

(UP) (ii) 16 seconds and 10 seconds ***(A1)(A1)***

Note: Accept 16.1 or 16.2 for the upper quartile value.

(iii) IQR $= 6$ seconds ***(A1)*(ft)** ***[4 marks]***

Note: **(ft)** from reasonable answers to (ii).

(b) value seen 650 ***(A1)***
2000 – value $= 2000 - 650$ ***(M1)***
$= 1350$ ***(A1)(G2)*** ***[3 marks]***

(c) 55 % of $2000 = 1100$ ***(A1)***
$p = 13.5$ ***(A1)(G2)*** ***[2 marks]***

(d) (i) $a = 500$ ***(A1)***

(ii) $b = 150$ ***(A1)*** ***[2 marks]***

(UP) (e) (i) $\bar{t} = 13.25$ seconds (13.3 seconds) ***(G2)***

OR

$$\bar{t} = \frac{7.5 \times 500 + 12.5 \times 850 + 17.5 \times \text{their } a + 22.5 \times \text{their } b}{2000}$$ ***(M1)***

(UP) $\bar{t} = 13.25$ seconds (13.3 seconds) ***(A1)*(ft)**

Note: Award **(ft)** from their *a* and their *b* only if working is seen.

(ii) $\sigma = 4.41$ seconds ***(G1)*** ***[3 marks]***

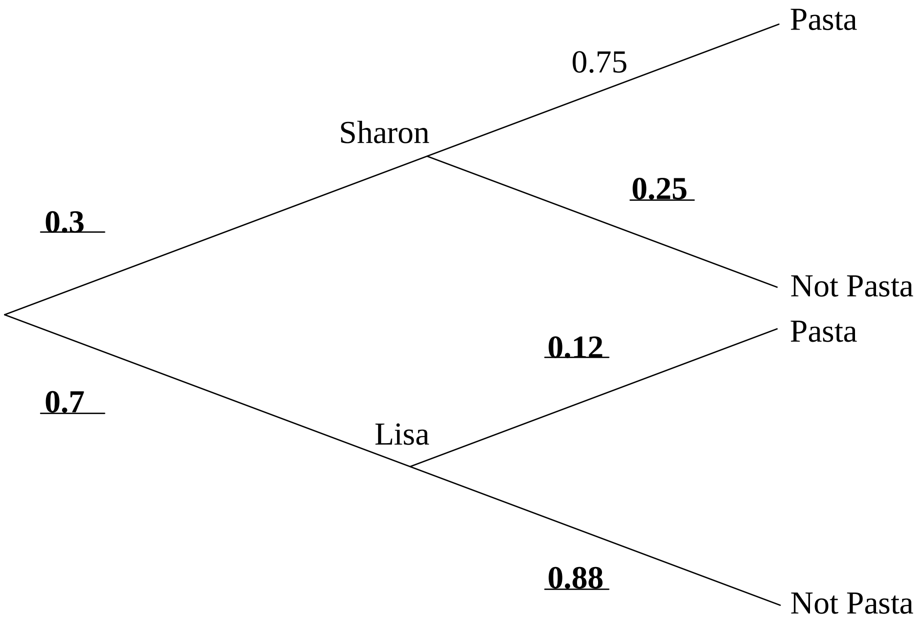
(f) $\bar{t} - \sigma = 8.84$ ***(A1)*(ft)**
Their $\bar{t} - \sigma$ compared to 9.5 ***(R1)***
Pedro does not receive the bonus ***(A1)*(ft)** ***[3 marks]***

Note: Do not award ***(R0)(A1)***.

Total [17 marks]

QUESTION 2

(i) (a)



> **Note:** Award ***(A1)*** for each correct pair.

(A3) ***[3 marks]***

(b) $0.7 \times 0.88 = 0.616 \ \left(\frac{77}{125}, 61.6\ \%\right)$ ***(M1)(A1)*(ft)*(G2)*** ***[2 marks]***

> **Note:** Award ***(M1)*** for multiplying the correct probabilities.

(c) $0.3 \times 0.25 + 0.7 \times 0.88$ ***(M1)(M1)***

> **Notes:** Award ***(M1)*** for a relevant two-factor product, could be $S \times NP$ **OR** $L \times NP$.
> Award ***(M1)*** for summing 2 two-factor products.

$\text{P} = 0.691 \ \left(\frac{691}{1000}, 69.1\ \%\right)$ ***(A1)*(ft)*(G2)*** ***[3 marks]***

> **Notes:** **(ft)** from their answer to (b).

(d) $\frac{0.616}{0.691}$ ***(M1)(A1)***

> **Note:** Award ***(M1)*** for substituted conditional probability formula, ***(A1)*** for correct substitution.

$\text{P} = 0.891 \ \left(\frac{616}{691}, 89.1\ \%\right)$ ***(A1)*(ft)*(G2)*** ***[3 marks]***

continued…

Question 2 continued

(ii) (a) 3 ***(A1)*** ***[1 mark]***

(b) For 5, 4, 7 (0) seen with no extra values ***(A1)***
16 ***(A1)(G2)*** ***[2 marks]***

(c) They like (both) the *Salseros (S)* **and** they like the *Bluers (B)* ***(A1)(A1)*** ***[2 marks]***

Note: Award ***(A1)*** for "and", ***(A1)*** for the correct groups.

(d) $R \cap B \cap S'$ ***(A1)(A1)*** ***[2 marks]***

Note: Award ***(A1)*** for $R \cap B$, ***(A1)*** for $\cap S'$

(e) (i) $21 + 3x = 33$ ***(M1)***
$x = 4$ ***(A1)(G2)***

(ii) 17 ***(A1)*(ft)** ***[3 marks]***

Total [21 marks]

QUESTION 3

(a) Gradient of CD $= \dfrac{1-(-1)}{-2-(-1)}$ ***(M1)***

$= -2$ ***(A1)(G2)*** ***[2 marks]***

Note: Award ***(M1)*** for correct substitution in gradient formula.

(b) Gradient of AD $= \dfrac{1}{2}$ ***(A1)***

$-2 \times \dfrac{1}{2} = -1$ or $\dfrac{1}{2}$ is negative reciprocal of –2 ***(M1)***

Hence AD is perpendicular for CD. ***(AG)*** ***[2 marks]***

Note: Last line must be seen for the ***(M1)*** to be awarded.

(c) $y = -2x - 3$ ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Award ***(A1)*****(ft)** for their (a), ***(A1)*****(ft)** for –3.
If part (a) incorrect award ***(A1)*****(ft)** for their y-intercept only if working is seen.

OR

$y - 1 = -2(x + 2)$ ***(A1)*****(ft)*****(A1)***

OR

$y + 1 = -2(x + 1)$ ***(A1)*****(ft)*****(A1)***

Note: Award ***(A1)*****(ft)** for their (a), ***(A1)*** for correct substitution of point.

$2x + y = -3$ ***(A1)*****(ft)** ***[3 marks]***

Note: The final ***(A1)*****(ft)** is for their equation in the stated form.

(d) $\mathrm{E}(-3, 3)$ (Accept $x = -3, y = 3$) ***(G2)***

OR

Award ***(M1)*** for solving the pair of simultaneous equations by hand. ***(A1)*****(ft)** for correct answer, **(ft)** from their (c). ***(M1)(A1)*****(ft)**

OR

Award ***(M1)*** for having extended the lines in their own graph seen drawn on answer paper. ***(A1)*** for correct answer. ***(M1)(A1)*** ***[2 marks]***

Note: Missing coordinate brackets receive ***(G1)(G0)*** or ***(M1)(A0)***.

continued…

Question 3 continued

(e) Distance between A and D $=\sqrt{4^2+2^2}$ ***(M1)***

$=\sqrt{20}$ **OR** $2\sqrt{5}$ **OR** 4.47 (3 *s.f.*) ***(A1)(G2)*** ***[2 marks]***

Note:	Award ***(M1)*** for correct substitution into the distance formula, ***(A1)*** for correct answer.

(f) Area of $\text{ADE}=\frac{1}{2}\sqrt{20}\times\sqrt{20}$ ***(M1)***

$=10$ ***(A1)*****(ft)*****(G2)*** ***[2 marks]***

Follow through from (e).

Total [13 marks]

QUESTION 4 ***Unit penalty (UP) applies in parts (a)(ii) (b) (c) and (e)***

(a) (i) 60° ***(A1)***

(ii) Area $= \dfrac{6 \times 6 \times \sin 60^\circ}{2}$ ***(M1)(A1)***

(UP) $= 15.6 \text{ cm}^2 \quad \left(9\sqrt{3}\right)$ ***(A1)*****(ft)*****(G2)*** ***[4 marks]***

> **Note:** Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct values. Accept alternative correct methods.

(b) Surface Area $= 15.58 \times 2 + 23 \times 6 \times 3$ ***(M1)(M1)***

> **Note:** Award ***(M1)*** for two terms with 2 and 3 respectively, ***(M1)*** for 23×6 (138).

(UP) Surface Area $= 445 \text{ cm}^2$ ***(A1)*****(ft)*****(G2)*** ***[3 marks]***

(c) weight $= 1.5 \times 15.59 \times 23$ ***(M1)(M1)***

> **Note:** Award ***(M1)*** for finding the volume, ***(M1)*** for multiplying their volume by 1.5.

(UP) weight $= 538$ g ***(A1)*****(ft)*****(G3)*** ***[3 marks]***

(d) $\cos\alpha = \dfrac{4^2 + 6^2 - 7^2}{2 \times 4 \times 6}$ ***(M1)(A1)***

> **Note:** Award ***(M1)*** for using cosine rule with values from the problem, ***(A1)*** for correct substitution.

$\alpha = 86.41\ldots$ ***(A1)***

$\alpha = 86.4^\circ$ ***(AG)*** ***[3 marks]***

> **Note:** $86.41\ldots$ must be seen for final ***(A1)*** to be awarded.

(e) $l \times \dfrac{4 \times 6 \times \sin 86.4^\circ}{2} \times 1.5 = 500$ ***(M1)(A1(M1)***

> **Notes:** Award ***(M1)*** for finding an expression for the volume, ***(A1)*** for correct substitution, ***(M1)*** for multiplying the volume by 1.5 and equating to 500, or for equating the volume to $\dfrac{500}{1.5}$.
> If formula for volume is not correct but consistent with that in (c) award at most ***(M1)(A0)*****(ft)*****(M1)(A0)***.

(UP) $l = 27.8$ cm ***(A1)(G3)*** ***[4 marks]***

Total [17 marks]

QUESTION 5

(a) $f'(x) = 3 - \frac{24}{x^3}$ ***(A1)(A1)(A1)*** ***[3 marks]***

Note: Award ***(A1)*** for 3, ***(A1)*** for –24, ***(A1)*** for $x^3 \left(\text{or } x^{-3}\right)$. If extra terms present award at most ***(A1)(A1)(A0)***.

(b) $f'(1) = -21$ ***(M1)(A1)*****(ft)*****(G2)*** ***[2 marks]***

Note: **(ft)** from their derivative only if working seen.

(c) Derivative (gradient, slope) is negative. Decreasing. ***(R1)(A1)*****(ft)** ***[2 marks]***

Note: Do not award ***(R0)(A1)***.

(d) $3 - \frac{24}{x^3} = 0$ ***(M1)***

$x^3 = 8$ ***(A1)***

$x = 2$ ***(A1)*****(ft)*****(G2)*** ***[3 marks]***

(e) (i) $(2, 9)$ (Accept $x = 2, y = 9$) ***(A1)(A1)(G2)***

Notes: **(ft)** from their answer in (d).
Award ***(A1)(A0)*** if brackets not included and not previously penalized.

(ii) 0 ***(A1)***

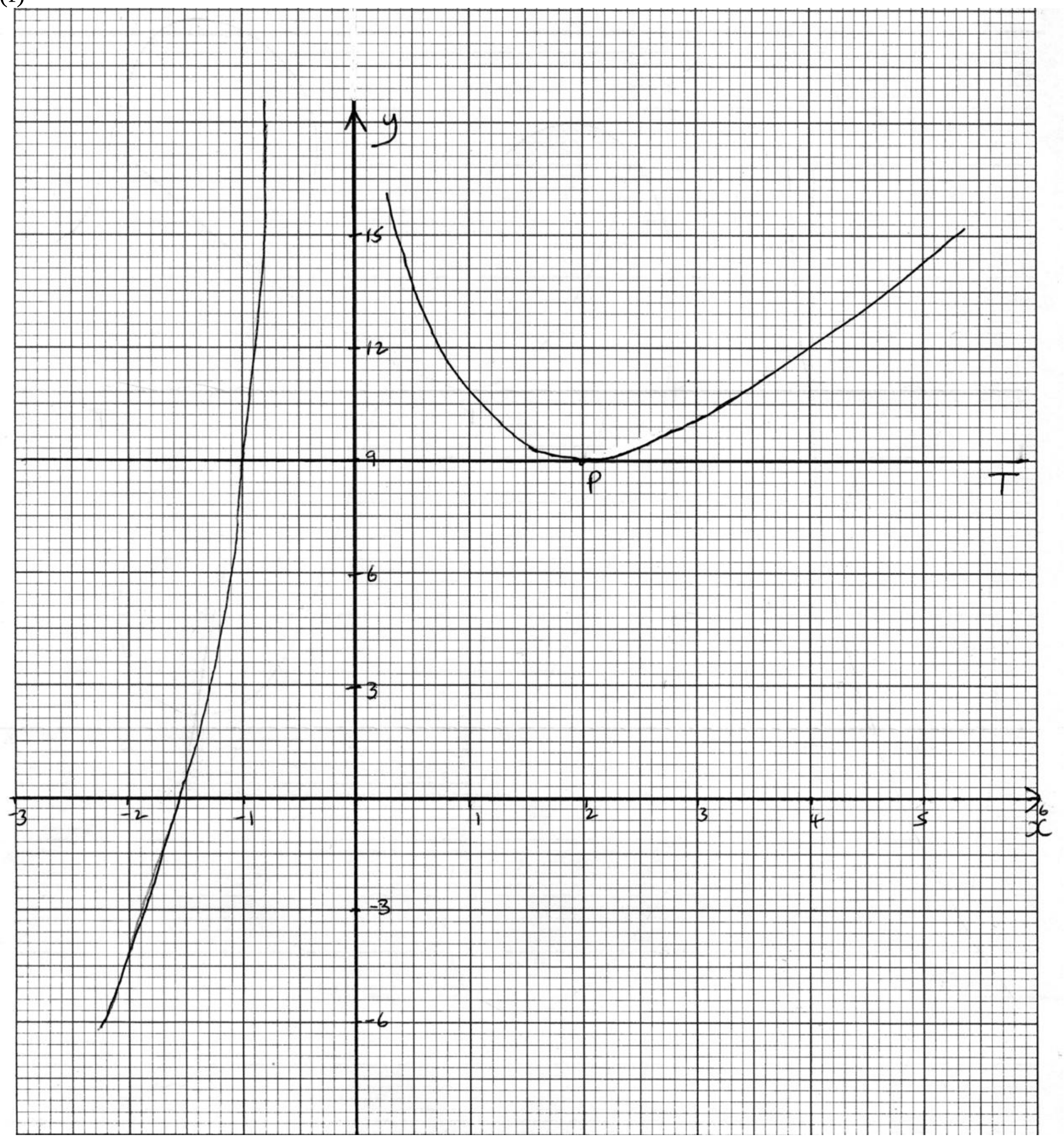
(iii) $y = 9$ ***(A1)(A1)*****(ft)*****(G2)*** ***[5 marks]***

Notes: Award ***(A1)*** for $y =$ constant, ***(A1)*** for 9.
Award ***(A1)*****(ft)** for their value of y in (e)(i).

continued …

Question 5 continued

(f)



(A4) ***[4 marks]***

Notes:	Award ***(A1)*** for labels and some indication of scale in the stated window. Award ***(A1)*** for correct general shape (curve must be smooth and must not cross the *y*-axis). Award ***(A1)*** for *x*-intercept seen in roughly the correct position. Award ***(A1)*** for minimum (P).

continued…

Question 5 continued

(g) (i) Tangent drawn at P (line must be a tangent and horizontal). ***(A1)***
Tangent labeled *T*. ***(A1)***

(ii) $x = -1$ ***(G1)*(ft)** ***[3 marks]***

Note:	**(ft)** from their tangent equation only if tangent is drawn and answer is consistent with graph.

Total [22 marks]